Notes from Fri 9 Sept.

I. Recap of One.I.3 thus far

- The number of solutions in the set of all solutions of a system of linear equations will be
  - none (zero)
  - exactly 1
  - an ∞ number

No way to get exactly 5 solns.

- Every system of linear equations will have a solution set with form

$$\{\vec{p} + \vec{h} : \vec{h} \text{ soln to } \underbrace{\text{homogeneous system}}\}$$

particular + homog. = general

all zero's

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n &= 0 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n &= 0 \\ &\vdots \\ a_{k1}x_1 + a_{k2}x_2 + \dots + a_{kn}x_n &= 0 \end{aligned}$$

$\vec{b}$ vector is all zeros.

$\vec{b}$ column

$$\vec{b} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

- Every system of linear equations will have a solution set with form

$$\{ \vec{p} + \vec{h} : \vec{h} \text{ soln. to homogeneous system} \}$$

Example of this ~~Thm in~~ action.

SoLE: $x + 2y - z = 2$
$2x - y - 2z + w = 5$

(i) Observe that $\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 12/5 \\ -1/5 \\ 0 \\ 0 \end{pmatrix}$ is a (particular) solution (ie a "$\vec{p}$")

(ii) Solve homogeneous system

$x + 2y - z = 0$
$2x - y - 2z + w = 0$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 0 & 0 \\ 2 & -1 & -2 & 1 & 0 \end{array}\right] \xrightarrow{-2e_1 + e_2 \mapsto e_2} \left[\begin{array}{cccc|c} 1 & 2 & -1 & 0 & 0 \\ 0 & -5 & 0 & 1 & 0 \end{array}\right]$$

$x + 2y - z = 0$
$-5y + w = 0$

So $y = w/5$

$x = -2y + z = \frac{-2}{5}w + z$

$$\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} \frac{-2}{5}w + z \\ w/5 \\ z \\ w \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} z + \begin{pmatrix} -2/5 \\ 1/5 \\ 0 \\ 1 \end{pmatrix} w$$

~~$\begin{pmatrix} w+z \\ w+5 \\ z \\ w \end{pmatrix}$~~

Solution to the Original SOLE:

$$\left\{ \underbrace{\begin{pmatrix} 12/5 \\ -1/5 \\ 0 \\ 0 \end{pmatrix}}_{\vec{p}} + \underbrace{\left[\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} z + \begin{pmatrix} -2/5 \\ 1/5 \\ 0 \\ 1 \end{pmatrix} w \right]}_{\vec{h}}, \; z, w \in \mathbb{R} \right\}$$

The last of One.I.2

definition: A square matrix is nonsingular

if it is the coefficient matrix

of a homogeneous system with a unique solution.

Otherwise, the matrix is singular.

Examples

	SoLE	matrix form	Coefficient matrix
$\boxed{A}$	$\begin{aligned} x+y&=7 \\ x+2y&=4 \end{aligned}$	$\left[\begin{array}{cc:c} 1 & 1 & 7 \\ 1 & 2 & 4 \end{array}\right]$	$\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$
$\boxed{B}$	$\begin{aligned} x+2y-z &= 2 \\ 2x-y-2z+w &= 5 \end{aligned}$	$\left[\begin{array}{cccc:c} 1 & 2 & -1 & 0 & 2 \\ 2 & -1 & -2 & 1 & 5 \end{array}\right]$	$\begin{bmatrix} 1 & 2 & -1 & 0 \\ 2 & -1 & -2 & 1 \end{bmatrix}$

Is $\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ nonsingular? Yes

Does the SoLE $\begin{aligned} x+y&=0 \\ x+2y&=0 \end{aligned}$ have a unique solution? Yes

Does $\begin{aligned} x+y&=0 \\ x+2y&=0 \end{aligned}$ have exactly 1 soln or ∞ # solns? Yes

row ops $\begin{aligned} x+y&=0 \\ y&=0 \end{aligned}$ $\Rightarrow$ $x=y=0$ ✓

geometrically, nonparallel lins. So intersect in $\underline{\underline{1}}$ point.

Let's make 3x3 matrices that are

① nonsingular

$$\begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \end{array} \quad e_3 - e_1 \to e_3 \quad \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \begin{array}{c} e_3 + e_2 \\ \downarrow \\ e_3 \end{array} \quad \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

② singular

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad 0x_1 + 0x_2 + 0x_3 = 0 \iff 0 = 0$$