Examples of Properties of the Determinant

Let $A = \begin{pmatrix} 2 & 0 & 5 \\ 0 & 1 & 3 \\ -1 & 0 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 8 & 5 \\ 0 & 1 & 3 \\ 0 & 0 & 4 \end{pmatrix}$

1. Find $det(A)$ by expanding along row 1 and along column 2.

$\det(A) = 2\begin{vmatrix} 1 & 3 \\ 0 & 4 \end{vmatrix} - 0\begin{vmatrix} \quad \end{vmatrix} + 5\begin{vmatrix} 0 & 1 \\ -1 & 0 \end{vmatrix} = 2(4) + 5(0-(-1)) = 8 + 5 = 13$

$\det(A) = -0\begin{vmatrix} \quad \end{vmatrix} + 1\cdot\begin{vmatrix} 2 & 5 \\ -1 & 4 \end{vmatrix} + 0\begin{vmatrix} \quad \end{vmatrix} = 8 - (-5) = 13$

2. Find $det(B)$ $= 2\cdot\begin{vmatrix} 1 & 3 \\ 0 & 4 \end{vmatrix} = 2\cdot 1\cdot 4 = 8 =$ prod of main diagonal terms.

$A \xrightarrow{r_1 \leftrightarrow r_3} \begin{pmatrix} -1 & 0 & 4 \\ 0 & 1 & 3 \\ 2 & 0 & 5 \end{pmatrix} \xrightarrow{2r_1 + r_3 \mapsto r_3} \begin{pmatrix} -1 & 0 & 4 \\ 0 & 1 & 3 \\ 0 & 0 & 13 \end{pmatrix} = C$

3. Let C be obtained from A by (i) exchanging rows 1 and 3 followed by (ii) adding 2*row 1 to row 3. Find $det(C)$.

$\det(C) = -13 = -\det(A)$

$\det(D) = 104$ ↑ technology

4. Find $D = BA$ and find $det(D)$ two ways.

$\det(B)\,d(A) = 13\cdot 8 = 104$

technology

$D = BA = \begin{pmatrix} 2 & 8 & 5 \\ 0 & 1 & 3 \\ 0 & 0 & 4 \end{pmatrix}\begin{pmatrix} 2 & 0 & 5 \\ 0 & 1 & 3 \\ -1 & 0 & 4 \end{pmatrix} = \begin{pmatrix} -1 & 8 & 54 \\ -3 & 1 & 15 \\ -4 & 0 & 16 \end{pmatrix}$