SECTION 2.3.2: DIMENSION

1. **Theorem 2.4** If V has a finite basis, then

   all bases of V have the same number of vectors.

All the questions below reference the vector space V with dimension n.

2. Let S be a set of linearly independent vectors from V. What can you say about the size of S and why?

   $|S| \le n$. If S had more vectors, then V would have bases w/ more than n vectors.

4. Let S be a set of vectors from V such that $span(S) = V$. What can you say about the size of S and why?

   $|S| \ge n$. If S has fewer vectors, then we could find a basis in S with fewer than n vectors.

1. Let S be a set of linearly independent vectors from V. Can you expand S into a basis? How?

   If S spans V, we're done. If not, pick $\vec{v} \in V$ where $\vec{v} \notin [S]$. Add $\vec{v}$. We know $S_1 = S \cup \{v\}$ is lin indep. Repeat on S_1.

3. Let S be a set of vectors from V such that $span(S) = V$. Can you construct a basis from S? How?

   If S is linearly dependent, we're done. If not, find a vector in S that is a linear combo of the others, say $\vec{s}$. Delete it: $S_1 = S - \{\vec{s}\}$ which must still span V. Repeat on S_1.

6. Assume S has exactly n vectors in it. What is the *least amount of work needed* to show S is a basis of V and why?

   Show S is lin. independent OR Show S spans V.

   If S is lin indepent and $|S| = n$, then S must span V.

   If S spans V and $|S| = n$, then S must be lin. independt.