1. **definition** The *row space* of a matrix is the span of the rows of A

   The *row rank* is the dimension of the row space

2. Lemma 3.4: Let A' be the reduced row echelon form of matrix A. The nonzero rows of A' are

   a basis for the row space of A.

   why? ① row space(A) = rowspace(A') since they are row equivalent.
   ② nonzero rows of A' are linearly independent.

3. **definition** The *column space* of a matrix is the span of columns of A

   The *column rank* is the dimension of column space of A.

4. **definition** The *transpose* of a matrix is the matrix obtained by switching rows & columns.

   col space A = row space A^T

5. Lemma 3.10: Row operations do not change Column rank.

6. Theorem: For any matrix, what is the relationship between row rank and column rank of matrices?
   They are equal! row rank(A) = col.rank(A)

7. **definition** The *rank* of a matrix is col rank or row rank.

8. Theorem: For linear systems with n unknowns and with coefficient matrix A the following statements are equivalent.
   - A has rank r
   - rref(A) has r leading 1's.
   - dim of solution space of $\left[A \,\vdots\, \begin{matrix}0\\0\\0\end{matrix}\right]$ is $n-r$.
     (i.e. rref(A) has n−r columns that do NOT have leading 1's)

9. Corollary: For the $n \times n$ matrix A, the following are equivalent.
   - A has rank n
   - A is nonsingular
   - rows are linearly independent
   - Columns are linearly independent
   - Any SoLE $([A \,\vdots\, \vec{b}])$ with A as a cofficient matrix has a unique solution.