SECTION 3.2.1 HOMOMORPHISMS/LINEAR MAPS

1. **Review** For vector spaces V and W, the function $f : V \to W$ is called an *isomorphism* if

2. **Review** For vector spaces V and W, the function $f : V \to W$ is called a *homomorphism* or *linear map* if

3. **Examples:**

   (a) $f : \mathcal{P}_2 \to \mathbb{R}^2$ defined as $f(ax^2 + bx + c) = \begin{pmatrix} a \\ b + c \end{pmatrix}$ (or $ax^2 + bx + c \mapsto \begin{pmatrix} a \\ b + c \end{pmatrix}$)

   (b) The *projection* $\pi : \mathbb{R}^3 \to \mathbb{R}^2$ defined by $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \mapsto \begin{pmatrix} x \\ y \end{pmatrix}$

   (c) The zero homomorphism $z : \mathbb{R}^4 \to \mathcal{M}_{2\times 2}$

4. Lemma 1.6:

5. Theorem 1.9:

6. Consequence of Theorem 1.9: Define a homomorphism h from $\mathcal{P}_2$ to $\mathcal{M}_{2\times 2}$ by defining h on a basis of $\mathcal{P}_2$ and demonstrating how h operates on an arbitrary element of $\mathcal{P}_2$.

7. Definition 1.12