1. Summary of our 3.2.2 Examples

   (a) $f:\mathbb{R}^2 \to \mathbb{R}$ defined as $f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = x+y$

   $\mathcal{R}(f) = \mathbb{R}$, $\text{rank}(f) = 1$, $\mathcal{N}(f) = f^{-1}(0) = \text{span}\left(\left\{\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right\}\right)$, $\text{nullity}(f) = 1$

   dimension of domain $= 2 = 1+1 = \text{rank}(f) + \text{nullity}(f)$

   (b) $f:\mathbb{R}^2 \to \mathbb{R}^3$ defined as $f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x \\ y \\ y \end{bmatrix}$

   $\mathcal{R}(f) = \text{span}\left(\left\{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}\right\}\right)$, $\text{rank}(f) = 2$, $\mathcal{N}(f) = f^{-1}(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}) = \left\{\begin{bmatrix} 0 \\ 0 \end{bmatrix}\right\}$ $\text{nullity}(f) = 0$

   dimension of domain $= 2 = 2+0 = \text{rank}(f) + \text{nullity}(f)$

2. (Theorem 2.14 and Corollary 2.17) Assume $f: V \to W$ is a linear map between vector spaces V and W.

   Then dimension (V) = rank f + nullity f

   Why? Say $\dim(V) = n$.

   ① Find basis for $\mathcal{N}(f)$ = null space in V. * If nullity=0, nothing to do here!!

   ② Extend basis of $\mathcal{N}(f)$ to basis for all of V.

   $B = \langle \underbrace{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_k}_{\text{f maps to } \vec{0}_W}, \underbrace{\vec{w}_{k+1}, \vec{w}_{k+2}, \ldots, \vec{w}_n}_{\text{form basis of } \mathcal{R}(f)} \rangle$

3. (Theorem 2.20) Assume $f: V \to W$ is a linear map between vector spaces V and W and $dim(V) = n$. The following are equivalent statements.

   ① f is a 1-1 map

   ② rank of f is n

   ③ nullity of f is 0

   ④ f^{-1} is a function or map from W to V. (It's the Calc style 'inverse'!)

   ⑤ If $\langle \vec{b}_1, \vec{b}_2, \ldots, \vec{b}_n \rangle$ is a basis of V, then

   $\langle f(\vec{b}_1), f(\vec{b}_2), \ldots, f(\vec{b}_n) \rangle$ is a basis of $\mathcal{R}(f)$

   range of f.