1. Summary of our 3.2.2 Examples

   (a) $f : \mathbb{R}^2 \to \mathbb{R}$ defined as $f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = x + y$

   $\mathcal{R}(f) = \mathbb{R},\ \text{rank}(f) = 1,\ \mathcal{N}(f) = f^{-1}(0) = \text{span}\left(\left\{\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right\}\right),\ \text{nullity}(f) = 1$
   dimension of domain $= 2 = 1 + 1 = \text{rank}(f) + \text{nullity}(f)$

   (b) $f : \mathbb{R}^2 \to \mathbb{R}^3$ defined as $f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x \\ y \\ y \end{bmatrix}$

   $\mathcal{R}(f) = \text{span}\left(\left\{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}\right\}\right),\ \text{rank}(f) = 2,\ \mathcal{N}(f) = f^{-1}(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}) = \left\{\begin{bmatrix} 0 \\ 0 \end{bmatrix}\right\}\ \text{nullity}(f) = 0$
   dimension of domain $= 2 = 2 + 0 = \text{rank}(f) + \text{nullity}(f)$

2. (Theorem 2.14 and Corollary 2.17) Assume $f : V \to W$ is a linear map between vector spaces V and W.

   Why?

3. (Theorem 2.20) Assume $f : V \to W$ is a linear map between vector spaces V and W and $dim(V) = n$. The following are equivalent statements.

   (a) The function f is a one-to-one map.

   (b) The rank of f is

   (c) The nullity of f is

   (d) The relation f^{-1} is

   (e) If $\langle \vec{b_1}, \vec{b_2}, \cdots \vec{b_n} \rangle$ forms a basis for V, then $\langle f(\vec{b_1}), f(\vec{b_2}), \cdots f(\vec{b_n}) \rangle$