SECTION 3.2.2 RANGE SPACE AND NULL SPACE

1. **Review** For vector spaces V and W, the function $f : V \to W$ is called a *homomorphism* or *linear map* if

2. (Some preliminary terminology) Assume $f : V \to W$, $S \subseteq V$, and $T \subseteq W$.

   image:

   inverse image:

   inverse function:

3. (Lemma 2.1, Definition 2.2, and some notation) Assume $f : V \to W$ is a linear map between vector spaces V and W and assume V' is a subspace of V.

4. (Lemma 2.10, Definition 2.11, and some notation) Assume $f : V \to W$ is a linear map between vector spaces V and W and assume W' is a subspace of W.

5. (Theorem 2.14 and Corollary 2.17) Assume $f : V \to W$ is a linear map between vector spaces V and W.

6. (Theorem 2.20) Assume $f : V \to W$ is a linear map between vector spaces V and W and $dim(V) = n$. The following are equivalent statements.

Examples:

1. $f : \mathbb{R}^2 \to \mathbb{R}$ defined as $f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = x + y$

   (a) Find the image of $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

   (b) Find the image of S where $S = \{\begin{bmatrix} x \\ 10 - x \end{bmatrix} \ : \ x \in \mathbb{R}\}$

   (c) Find the inverse image of

   (d) Find the range space of f and determine its rank.

   (e) Find the null space and nullity of f.

2. $f : \mathbb{R}^2 \to \mathbb{R}^3$ defined as $f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x \\ y \\ y \end{bmatrix}$

(a) Find the image of $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

(b) Find the image of S where $S = \{\begin{bmatrix} 0 \\ y \end{bmatrix} : y \in \mathbb{R}\}$

(c) Find the inverse image of

(d) Find the range space of f and determine its rank.

(e) Find the null space and nullity of f.