SECTION 3.3.1: REPRESENTING LINEAR MAPS WITH MATRICES

1. The Big Idea: A linear map between vector spaces can always be described as a matrix which can be used to find the image of vectors using the matrix-vector product. (a thinking-free automation)

2. The Big Idea in a formal way:
   Let $f : V \to W$ be a **linear map** between **vector spaces** V and W with **bases** $B = \langle \vec{b_1}, \vec{b_2}, \cdots, \vec{b_n} \rangle$, (for V of dimension n) and $D = \langle \vec{d_1}, \vec{d_2}, \cdots, \vec{d_m} \rangle$, (for W of dimension m).

   Then the matrix M representing the linear map has dimensions

   with columns formed as follows

   The image of a vector $\vec{v} \in V$ can be found by

3. Fact we will use: Any linear map $f : V \to W$ between vector spaces can be determined by

4. Example: $f : \mathbb{R}^3 \to \mathbb{R}^2$

bases: $B = \left\langle \vec{b_1} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \vec{b_2} = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}, \vec{b_3} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right\rangle$ and $D = \left\langle \vec{d_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \vec{d_2} = \begin{pmatrix} -1 \\ -1 \end{pmatrix} \right\rangle$

image of basis vectors: $f(\vec{b_1}) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, $f(\vec{b_2}) = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$, $f(\vec{b_3}) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

(a) Find the image of $\vec{v} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ under f and express its image with respect to the basis D **via actual thinking...**

(b) Find the image of $\vec{v} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ under f and express its image with respect to the basis D **via automation.**