Section 3.3.2: Any Matrix Represents a Linear Map

1. The Big Idea from 3.3.1: A linear map between vector spaces can always be described as a matrix which can be used to find the image of vectors using the matrix-vector product. (a thinking-free automation)

2. Review Example: $h : \mathbb{R}^3 \to \mathbb{R}^2$ by $h\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right) = \begin{bmatrix}0\\1\end{bmatrix}$, $h\left(\begin{bmatrix}0\\1\\0\end{bmatrix}\right) = \begin{bmatrix}-1\\0\end{bmatrix}$, $h\left(\begin{bmatrix}1\\1\\1\end{bmatrix}\right) = \begin{bmatrix}0\\0\end{bmatrix}$. Assume the basis for $\mathbb{R}^3$ is $B = \langle \begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}1\\1\\1\end{bmatrix} \rangle$ and $\mathcal{E}_2$ for $\mathbb{R}^2$. Find $\text{rep}_{B,\mathcal{E}_2}(h)$ and use it to find the image of $\vec{v} = \begin{bmatrix}1, 2, 3\end{bmatrix}$.

3. The Big Idea from 3.3.2: Given any $m \times n$ matrix M, we can view M as a linear map between two vector spaces $V \to W$ of dimensions with respect to any pair of bases.

4. Simple Example: Let $M = \begin{bmatrix}1 & 3\\0 & -1\\1 & 1\\2 & 3\end{bmatrix}$.

5. Let M be an $m \times n$ matrix representing the linear map $h : V \to W$, for vector spaces V and W of dimensions n and m respectively. (There is an underlying assumption that bases for V and W are known.)

   (a) **Theorem 2.4:** Rank of $M =$ rank of h

   **Revisit example in #4

   (b) **Corollary 2.6**

   - h is onto if and only if rank of M is
   - h is one-to-one if and only if rank of M is

   (c) **Lemma 2.9:** h is an isomorphism if and only of M is

6. Give examples of singular and nonsingular homomorphisms from $\mathbb{R}^3 \to \mathbb{R}^3$.