1. **Definition 1.3** Let A and B be $m \times n$ matrices, $r \in \mathbb{R}$. Then the operations of scalar multiplication $(r \cdot A)$ and matrix addition $(A + B)$ are performed componentwise.

2. **Example:** Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 3 & -1 \\ 4 & 1 & 1 \end{bmatrix}$. Find $10 \cdot A$ and $A + B$.

$$10A = 10 \cdot \begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 10 & -10 & 20 \\ 20 & 0 & 50 \end{bmatrix}, \quad A+B = \begin{bmatrix} 1+0 & -1+3 & 2-1 \\ 2+4 & 0+1 & 5+1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 6 & 1 & 6 \end{bmatrix}$$

3. **Thm 1.4:** If $f : V \to W$ and $g : V \to W$ are linear maps such that $rep_{B,B'}(f) = A$ and $rep_{B,B'}(g) = C$, then
   - the linear map $r \cdot f$ where r is a scalar can be represented as $r \cdot A$
   - the linear map $f + g$ can be represented as $A + B$

4. **Example:** Let f and g be linear maps from $\mathbb{R}^2$ to $\mathbb{R}^2$ such that their representation with respect to $\mathcal{E}_2$ is given by $A = \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 3 \\ 4 & 1 \end{bmatrix}$. Find $(f+g)\left(\begin{bmatrix} 2 \\ -5 \end{bmatrix}\right)$ two ways:

   (a) Finding $f(\begin{bmatrix} 2 \\ -5 \end{bmatrix})$ and $g(\begin{bmatrix} 2 \\ -5 \end{bmatrix})$ and adding the resulting vectors

$$f\left(\begin{bmatrix} 2 \\ -5 \end{bmatrix}\right) = \overset{A}{\begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}} \cdot \begin{bmatrix} 2 \\ -5 \end{bmatrix} = \begin{bmatrix} 2+5 \\ 4+0 \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \end{bmatrix}$$

$$g\left(\begin{bmatrix} 2 \\ -5 \end{bmatrix}\right) = \begin{bmatrix} 0 & 3 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ -5 \end{bmatrix} = \begin{bmatrix} 0-15 \\ 8-5 \end{bmatrix} = \begin{bmatrix} -15 \\ 3 \end{bmatrix}$$

So $f\left(\begin{bmatrix} 2 \\ -5 \end{bmatrix}\right) + g\left(\begin{bmatrix} 2 \\ -5 \end{bmatrix}\right)$

$$= \begin{bmatrix} 7 \\ 4 \end{bmatrix} + \begin{bmatrix} -15 \\ 3 \end{bmatrix} = \begin{bmatrix} -8 \\ 7 \end{bmatrix}$$

   (b) Finding $A + B = C$ and using C.

$$C = A+B = \begin{bmatrix} 1+0 & -1+3 \\ 2+4 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 6 & 1 \end{bmatrix}$$

The same!

$$(f+g)\left(\begin{bmatrix} 2 \\ -5 \end{bmatrix}\right) = \begin{bmatrix} 1 & 2 \\ 6 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ -5 \end{bmatrix} = \begin{bmatrix} 1 \cdot 2 + 2(-5) \\ 6 \cdot 2 + 1(-5) \end{bmatrix} = \begin{bmatrix} -8 \\ 7 \end{bmatrix}$$