SECTION 3.4.1: SUMS AND SCALAR PRODUCTS

1. **Definition 1.3** Let A and B be $m \times n$ matrices, $r \in \mathbb{R}$. Then the operations of scalar multiplication $(r \cdot A)$ and matrix addition $(A + B)$ are performed componentwise.

2. **Example:** Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 3 & -1 \\ 4 & 1 & 1 \end{bmatrix}$. Find $10 \cdot A$ and $A + B$.

3. **Thm 1.4:** If $f : V \to W$ and $g : V \to W$ are linear maps such that $rep_{B,B'}(f) = A$ and $rep_{B,B'}(g) = C$, then
   - the linear map $r \cdot f$ where r is a scalar can be represented as
   - the linear map $f + g$ can be represented as

4. **Example:** Let f and g be linear maps from $\mathbb{R}^2$ to $\mathbb{R}^2$ such that their representation with respect to $\mathcal{E}_2$ is given by $A = \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 3 \\ 4 & 1 \end{bmatrix}$. Find $(f + g)\left(\begin{bmatrix} 2 \\ -5 \end{bmatrix}\right)$ two ways:

   (a) Finding $f(\begin{bmatrix} 2 \\ -5 \end{bmatrix})$ and $g(\begin{bmatrix} 2 \\ -5 \end{bmatrix})$ and adding the resulting vectors

   (b) Finding $A + B = C$ and using C.