

SECTION 3.4.2 AND 3.4.3: COMPOSITION OF LINEAR MAPS AND MATRIX MULTIPLICATION
(DAY 2)

1. **Take-aways from Monday** Let $f : V \rightarrow W$ and $g : W \rightarrow Y$ be linear maps with matrix representations A and B respectively. Then,

- the matrix representation of $(g \circ f) : V \rightarrow Y$ is BA with dimension $\dim(Y) \times \dim(V)$
- the function $(g \circ f)$ is a **linear map**. (ie the composition of linear maps is also linear!)
- Function composition - matrix multiplication **is NOT** commutative.
- Function composition - matrix multiplication **IS** associative.

$$(f \circ g \circ h) = f \circ (g \circ h) = (f \circ g) \circ h$$

$$ABC = A(BC) = (AB)C$$

- Function composition - matrix multiplication **is** distributive.

$$f \circ (g+h) = (f \circ g) + (f \circ h)$$

$$A(B+C) = AB + AC$$

Note f, g, h being linear maps really matters here!

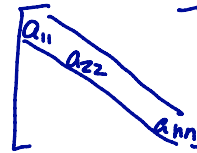
- $\sin(e^x + x^2) \neq \sin(e^x) + \sin(x^2)$
- $5(8x + 7x) = 5(8x) + 5(7x)$

2. Terminology

(a) main diagonal

A is square $n \times n$ matrix

main diagonal: a_{ii} entries



(b) identity matrix

I_n has 1's on main diagonal, 0's elsewhere

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(c) diagonal matrix

has 0's off the main diagonal

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -3 \end{bmatrix}, \begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 4 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 4 & 2 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 8 & 4 \\ 3 & -9 \end{bmatrix}$$

If D_n is diagonal and A is $n \times m$, what does $D_n A$ look like? $A D_n$?

Row i of A is multiplied by d_{ii}

(d) permutation matrix

has exactly one 1 in every row + every column and zeros elsewhere.

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, I_3 \quad \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = \begin{bmatrix} 4 & 5 & 6 \\ 1 & 2 & 3 \\ 7 & 8 & 9 \end{bmatrix}$$

If P is $n \times n$ permutation matrix, A is $n \times m$ matrix, then

PA permutes the rows of A .

AP ? permutes columns

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 3 \\ 5 & 4 & 6 \\ 8 & 7 & 9 \end{bmatrix}$$

(e) elementary (reduction) matrices

matrices that perform elementary row operations.

Ex: Construct M that adds $2 \times$ row 1 to row 2 + leaves everything else fixed

$$M \quad A \quad \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 6 & 9 & 12 \\ 7 & 8 & 9 \end{bmatrix}$$