

## SECTION 3.4.2 AND 3.4.3: COMPOSITION OF LINEAR MAPS AND MATRIX MULTIPLICATION

1. **Example** Let  $f : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  and  $g : \mathbb{R}^2 \rightarrow \mathbb{R}^4$  be linear maps with matrix representations

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 1 \\ 0 & 0 \\ 0 & 1 \\ 0 & 2 \end{bmatrix} \text{ (with standard bases } \mathcal{E}_3, \mathcal{E}_2, \text{ and } \mathcal{E}_4\text{.) Let } \vec{v} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}.$$

Find  $(g \circ f)(\vec{v})$ .

2. **Definition 2.3 of Matrix Multiplication**  $A = [a_{ij}]$  is  $m \times n$  and  $B = [b_{ij}]$  is  $n \times p$ . Then  $C = [c_{ij}] = AB$  is defined as

$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \cdots + a_{in}b_{nj}.$$

3. **Example:** Let  $A = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix}$ ,  $B = \begin{bmatrix} 2 & -1 \\ 1 & 3 \\ 1 & 0 \end{bmatrix}$ ,  $C = \begin{bmatrix} 10 & 0 & 4 \\ 0 & 3 & -1 \end{bmatrix}$ . Find the following products or state that they are undefined.

(a)  $AB$

(b)  $BA$

(c)  $A^2$

(d)  $BC$

(e)  $CB$

4. Observations: