1. (S 2.3.1) Recall $\mathcal{E}_3 = \left\langle \vec{e_1} = \begin{pmatrix}1\\0\\0\end{pmatrix}, \vec{e_2} = \begin{pmatrix}0\\1\\0\end{pmatrix}, \vec{e_3} = \begin{pmatrix}0\\0\\1\end{pmatrix} \right\rangle$. Let $B = \left\langle \vec{b_1} = \begin{pmatrix}1\\-1\\0\end{pmatrix}, \vec{b_2} = \begin{pmatrix}1\\1\\-2\end{pmatrix}, \vec{b_3} = \begin{pmatrix}1\\1\\1\end{pmatrix} \right\rangle$.

$\frac{1}{2}$ $\frac{1}{6}$ $\frac{1}{3} = \begin{matrix}1\\0\\0\end{matrix}$ check ✓

Let $\vec{v} = \begin{pmatrix}1\\-2\\3\end{pmatrix}_{\mathcal{E}_3}$. Find each representation below. ← We kind of assume this...

(a) $\operatorname{rep}_{\mathcal{E}_3}(\vec{v}) = \begin{pmatrix}1\\-2\\3\end{pmatrix}$

easy (e) $\operatorname{rep}_{\mathcal{E}_3}(\vec{b_3}) = \begin{pmatrix}1\\1\\1\end{pmatrix}$

N.t. Solve

$$c_1\begin{pmatrix}1\\-1\\0\end{pmatrix} + c_2\begin{pmatrix}1\\1\\-2\end{pmatrix} + c_3\begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}1\\-2\\3\end{pmatrix}$$

Don't re-invent the wheel!

(b) $\operatorname{rep}_B(\vec{v}) = \begin{pmatrix}3/2\\-7/6\\2/3\end{pmatrix}_B$

$$\left[\begin{array}{ccc:c}1&1&1&1\\-1&1&1&-2\\0&-2&1&3\end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc:c}1&0&0&3/2\\0&1&0&-7/6\\0&0&1&2/3\end{array}\right]$$

(f) $\operatorname{rep}_B(\vec{e_1}) = \begin{pmatrix}1/2\\1/6\\1/3\end{pmatrix}_B$

$$\left[\begin{array}{ccc:c}1&1&1&1\\-1&1&1&0\\0&-2&1&0\end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{c:c}I_3&\begin{matrix}1/2\\1/6\\1/3\end{matrix}\end{array}\right]$$

(c) $\operatorname{rep}_{\mathcal{E}_3}(\vec{b_1}) = \begin{pmatrix}1\\-1\\0\end{pmatrix}$

(g) $\operatorname{rep}_B(\vec{e_2}) = \begin{pmatrix}-1/2\\1/6\\1/3\end{pmatrix}_B$

$$\left[\begin{array}{ccc:c}1&1&1&0\\-1&1&1&1\\0&2&1&0\end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{c:c}I_3&\begin{matrix}-1/2\\1/6\\1/3\end{matrix}\end{array}\right]$$

easy

(d) $\operatorname{rep}_{\mathcal{E}_3}(\vec{b_2}) = \begin{pmatrix}1\\1\\-2\end{pmatrix}$

(h) $\operatorname{rep}_B(\vec{e_3}) = \begin{pmatrix}0\\-1/3\\1/3\end{pmatrix}_B$

$$\left[\begin{array}{ccc:c}1&1&1&0\\-1&1&1&0\\0&2&1&1\end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{c:c}I_3&\begin{matrix}0\\-1/3\\1/3\end{matrix}\end{array}\right]$$

Note: Do (f), (g), (h) simultaneously

$$\left[\begin{array}{ccc:ccc}1&1&1&1&0&0\\-1&1&1&0&1&0\\0&2&1&0&0&1\end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{c:ccc}I_3&1/2&-1/2&0\\&1/6&1/6&-1/3\\&1/3&1/3&1/3\end{array}\right]$$

2. (S 3.3.1) Define $h : \mathbb{R}^3 \to \mathbb{R}^3$ by $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \mapsto \begin{pmatrix} y+z \\ x+z \\ x+y \end{pmatrix}$ assuming $\mathcal{E}_3$ basis for domain + codomain.

(a) Find the matrix representation of h, $\mathrm{Rep}_{\mathcal{E}_3,\mathcal{E}_3}(h)$. $= A$

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \mapsto \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \qquad \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \mapsto \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \qquad \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \mapsto \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

So $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

(b) Find $h(\vec{v})$ for $\vec{v} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}_{\mathcal{E}_3}$.

$$A\vec{v} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} -2+3 \\ 1+3 \\ 1-2 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ -1 \end{bmatrix}$$

or $\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} \mapsto \begin{pmatrix} -2+3 \\ 1+3 \\ 1-2 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix}$

3. (S 3.3.1)

(a) Find $\mathrm{Rep}_{B,B}(h)$.

Way 3: Find $\mathrm{Rep}_{B,B}(h) = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

§ 3.5.1 + 3.5.2 How to get this.

(b) Find $\mathrm{Rep}_B(h(\vec{v}))$ for $\vec{v} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}_{\mathcal{E}_3}$.

Way 1: From 2a, $h(\vec{v}) = \begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix}$

Solve

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 4 \\ 0 & -2 & 1 & -1 \end{array}\right] \xRightarrow{\text{rref}} \left[\begin{array}{c|c} I_3 & \begin{matrix} -3/2 \\ 7/6 \\ 4/3 \end{matrix} \end{array}\right]$$

So $\mathrm{Rep}_B(h(\vec{v})) = \begin{pmatrix} -3/2 \\ 7/6 \\ 4/3 \end{pmatrix}$

whoa! Go back and look at $(\vec{v})_B$!

It's $\begin{pmatrix} 3/2 \\ -7/6 \\ 2/3 \end{pmatrix}$

Way 2: $B\vec{v}$ for appropiate matrix B...

$$\mathrm{Rep}_{\mathcal{E}_3,B}(h) = \begin{bmatrix} | & | & | \\ h(\vec{e}_1)_B & h(\vec{e}_2)_B & h(\vec{e}_3)_B \\ | & | & | \end{bmatrix}$$

$$= \begin{bmatrix} -1/2 & 1/2 & 0 \\ -1/6 & -1/6 & 1/3 \\ 2/3 & 2/3 & 2/3 \end{bmatrix} = B$$

$$B \cdot \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} = \begin{pmatrix} -3/2 \\ 7/6 \\ 4/3 \end{pmatrix} \checkmark$$