Section 3.5.1 and 3.5.2 Change of Basis
Motivating Example

1. (S 2.3.1) Recall $\mathcal{E}_3 = \left\langle \vec{e_1} = \begin{pmatrix}1\\0\\0\end{pmatrix}, \vec{e_2} = \begin{pmatrix}0\\1\\0\end{pmatrix}, \vec{e_3} = \begin{pmatrix}0\\0\\1\end{pmatrix} \right\rangle$. Let $B = \left\langle \vec{b_1} = \begin{pmatrix}1\\-1\\0\end{pmatrix}, \vec{b_2} = \begin{pmatrix}1\\1\\-2\end{pmatrix}, \vec{b_3} = \begin{pmatrix}1\\1\\1\end{pmatrix} \right\rangle$.

   Let $\vec{v} = \begin{pmatrix}1\\-2\\3\end{pmatrix}$. Find each representation below.

   (a) $\text{rep}_{\mathcal{E}_3}(\vec{v}) =$

   (b) $\text{rep}_{B}(\vec{v}) =$

   (c) $\text{rep}_{\mathcal{E}_3}(\vec{b_1}) =$

   (d) $\text{rep}_{\mathcal{E}_3}(\vec{b_2}) =$

   (e) $\text{rep}_{\mathcal{E}_3}(\vec{b_3}) =$

   (f) $\text{rep}_{B}(\vec{e_1}) =$

   (g) $\text{rep}_{B}(\vec{e_2}) =$

   (h) $\text{rep}_{B}(\vec{e_3}) =$

2. (S 3.3.1) Define $h : \mathbb{R}^3 \to \mathbb{R}^3$ by $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \mapsto \begin{pmatrix} y+z \\ x+z \\ x+y \end{pmatrix}$ assuming ______________________________

(a) Find the matrix representation of h, $\text{Rep}_{\mathcal{E}_3,\mathcal{E}_3}(h)$.

(b) Find $h(\vec{v})$ for $\vec{v} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}_{\mathcal{E}_3}$.

3. (S 3.3.1)

(a) Find $\text{Rep}_B(h(\vec{v}))$ for $\vec{v} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}_{\mathcal{E}_3}$.

(b) Find $\text{Rep}_{B,B}(h)$.