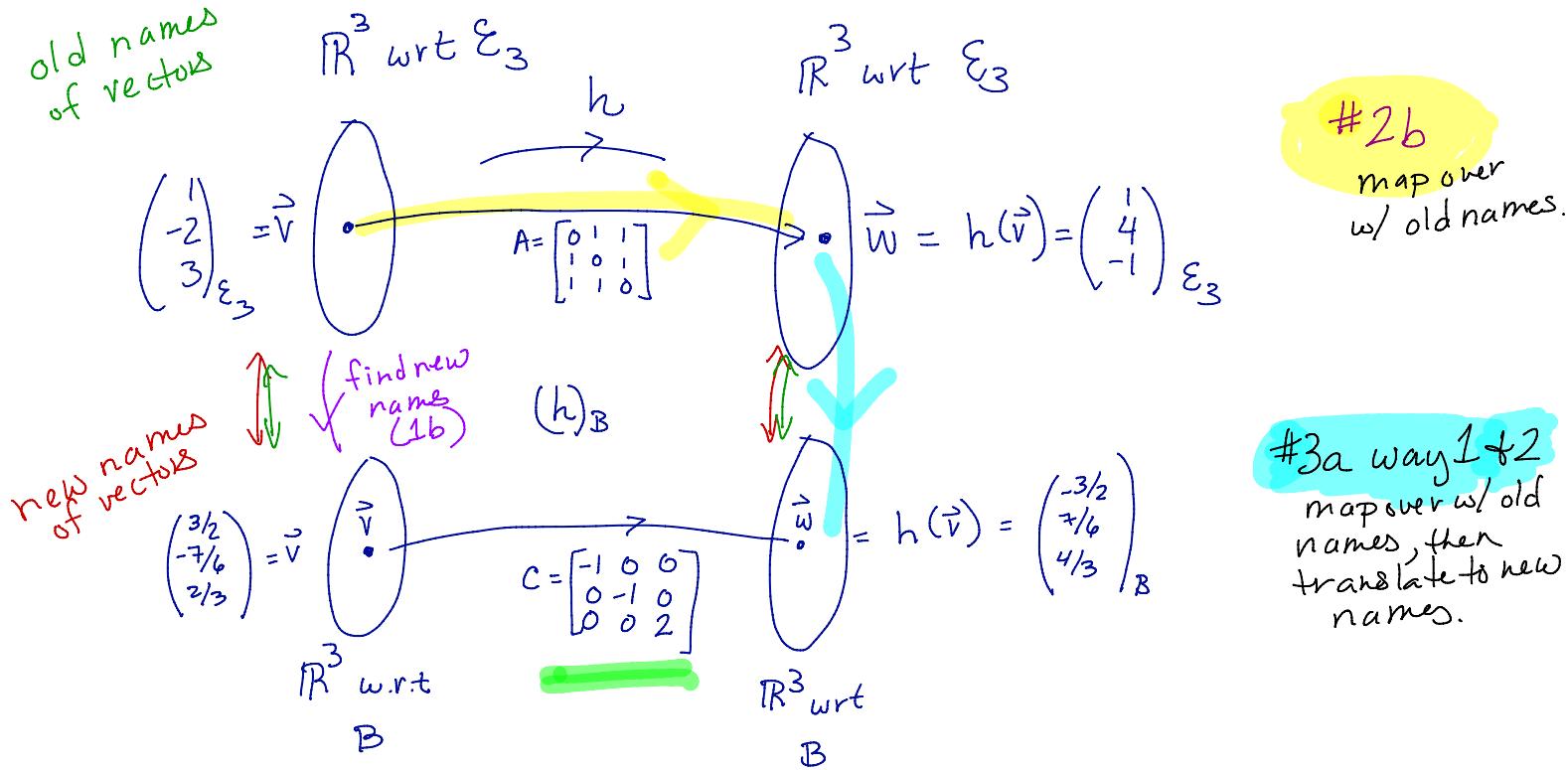


SECTION 3.5.1 AND 3.5.2 CHANGE OF BASIS
OBSERVATIONS FROM MONDAY'S MOTIVATING EXAMPLE

Example:

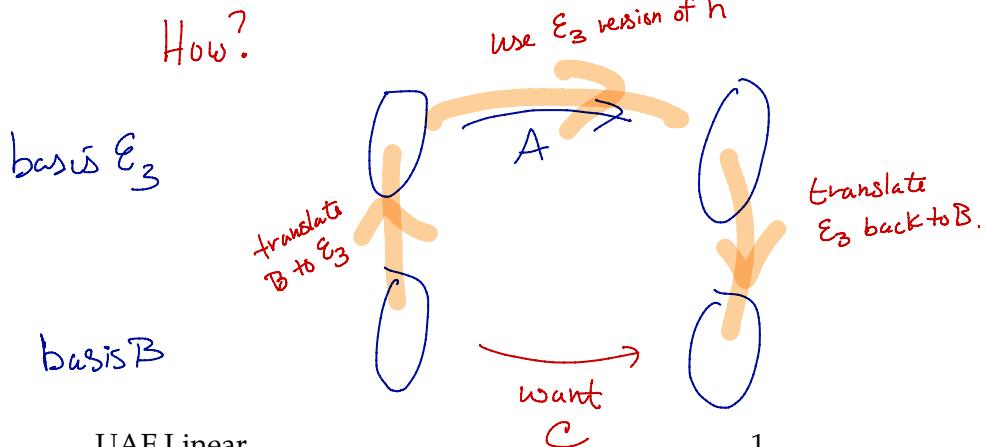
$$\mathcal{E}_3 = \left\langle \vec{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \vec{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \vec{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\rangle \quad B = \left\langle \vec{b}_1 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \vec{b}_2 = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}, \vec{b}_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\rangle$$

$$h : \mathbb{R}^3 \rightarrow \mathbb{R}^3 \text{ defined by } \begin{pmatrix} x \\ y \\ z \end{pmatrix} \mapsto \begin{pmatrix} y+z \\ x+z \\ x+y \end{pmatrix} \text{ w.r.t } \mathcal{E}_3$$



But, if we committed to just using B (in both domain and codomain), the matrix representation is simple!

How?



$$\begin{aligned} \text{undo } B \text{ to } \mathcal{E}_3 &= \mathcal{E}_3 \text{ to } B = D^{-1} \\ A &\quad B \text{ to } \mathcal{E}_3 = D \\ C &= \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & 0 \\ \frac{1}{6} & \frac{1}{6} & -\frac{1}{6} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \\ 0 & -2 & 1 \end{bmatrix} \\ D^{-1} - A \cdot D & \end{aligned}$$