

SECTION 3.5.1 AND 3.5.2 CHANGE OF BASIS
DO AN EXAMPLE ON YOUR OWN

Example:

$$\mathcal{E}_3 = \left\langle \vec{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \vec{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \vec{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\rangle \quad B = \left\langle \vec{b}_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \vec{b}_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \vec{b}_3 = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \right\rangle$$

$$h : \mathbb{R}^3 \rightarrow \mathbb{R}^3 \text{ has matrix representation } A = \begin{pmatrix} 5 & -3 & -6 \\ 3 & -1 & -6 \\ 3 & -3 & -4 \end{pmatrix} \text{ w.r.t } \mathcal{E}_3$$

1. Find $S = \text{Rep}_{B, \mathcal{E}_3}(id)$, the matrix that translates vectors in base B to vectors in \mathcal{E}_3 .

2. Find $T = \text{Rep}_{\mathcal{E}_3, B}(id)$, the matrix that translates vectors in base \mathcal{E}_3 to vectors in base B .

3. Find $C = \text{Rep}_{B, B}(h)$, the matrix representation of h with respect to basis B .

4. Check that your answers are correct by mapping the vector $\vec{v} = \begin{pmatrix} 2 \\ -2 \\ 6 \end{pmatrix}_{\mathcal{E}_3}$ using matrix A and using matrix C and showing that those output vectors are the same.

5. Draw a “potato picture” of what the matrices A , S , T , and C represent.