# SECTION ONE.I.2: DESCRIBING THE SOLUTIONS SET (AKA AESTHETICS)

Goals: (1) Reframe SoLE (and their solutions) in terms of matrices (vectors), (2) Review elementary vector notation and operations.

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How we solved a SoLE in Section One.I.1

$$\begin{cases} x_1 - 2x_2 + x_3 = 0 \\ 2x_2 - 8x_3 = 8 \\ 5x_1 - 5x_3 = 10 \end{cases} \xrightarrow[\rho_3 - 5\rho_1 \mapsto \rho_3]{} \begin{cases} x_1 - 2x_2 + x_3 = 0 \\ 2x_2 - 8x_3 = 8 \\ 10x_2 - 10x_3 = 10 \end{cases} \xrightarrow[\rho_3 - 5\rho_2 \mapsto \rho_3]{} \begin{cases} x_1 - 2x_2 + x_3 = 0 \\ 2x_2 - 8x_3 = 8 \\ 30x_3 = -30 \end{cases}$$

Conclude: $x_3 = -1,\ x_2 = 0,\ x_1 = 1$ via back substitution.

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How we solved a SoLE in Section One.I.2

$$\begin{cases} x_1 - 2x_2 + x_3 = 0 \\ 2x_2 - 8x_3 = 8 \\ 5x_1 - 5x_3 = 10 \end{cases} \qquad\qquad \begin{cases} x_1 - 2x_2 + x_3 = 0 \\ 2x_2 - 8x_3 = 8 \\ 30x_3 = -30 \end{cases} \quad \text{Solve as before.}$$

$$\Downarrow \qquad\qquad\qquad\qquad \Uparrow$$

$$\begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 2 & 8 & 8 \\ 5 & 0 & -5 & 10 \end{bmatrix} \xrightarrow[\rho_3 - 5\rho_1 \mapsto \rho_3]{} \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 2 & 8 & 8 \\ 0 & 10 & -10 & 10 \end{bmatrix} \xrightarrow[\rho_3 - 5\rho_2 \mapsto \rho_3]{} \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 2 & 8 & 8 \\ 0 & 0 & 30 & -30 \end{bmatrix}$$

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Example 1: Solve the SoLE $\begin{cases} w + y + 2z = 0 \\ w + 2x + y + 6z = 8 \\ -w + 2x + 2y + 2z = 20 \end{cases}$ by converting to matrices.

Vector Review

- vector versus scalar?
- vector addition:

  (requires the same dimensions!)
- scalar multiplication:

Return to Example 1. Write its solution in vector form.

Example 2: Assume that a SoLE has a matrix echelon form $A = \begin{bmatrix} 1 & 2 & 0 & 1 & 1 & 5 \\ 0 & 2 & -4 & 0 & 2 & -6 \\ 0 & 0 & 0 & 0 & 1 & 4 \end{bmatrix}$. Find the solution set of the SoLE.