Definition: A linear combination of $x_1, x_2, \cdots x_n$ is

$$c_1 x_1 + c_2 x_2 + c_3 x_3 + \dots + c_n x_n$$

where $c_i \in \mathbb{R}$.

Definition: A linear combination of vectors $\overline{u}_1, \overline{u}_2, \cdots \overline{u}_n$ is

$$c_1 \vec{u}_1 + c_2 \vec{u}_2 + \dots + c_n \vec{u}_n \quad \text{where } c_i \in \mathbb{R}$$

Example: Write three distinct linear combination of the vectors $\overline{u}_1 = (1, 2, 3)$ and $\overline{u}_2 = (1, -1, 1)$.

Example: Is $\overline{v} = (2, -1, 2)$ a linear combination of $\overline{u}_1 = (1, 2, 3)$ and $\overline{u}_2 = (1, -1, 1)$?

$$2\vec{u}_1 + 3u_2 = 2(1,2,3) + 3(1,-1,1) = (2,4,6) + (3,-3,3) = \underline{(5,1,9)}$$

$$1\vec{u}_1 + 0\vec{u}_2 = (1,2,3)$$

$$\sqrt{2}\,\vec{u}_1 - \pi\vec{u}_2 = \sqrt{2}(1,2,3) - \pi(1,-1,1) = (\sqrt{2}, 2\sqrt{2}, 3\sqrt{2}) - (\pi, -\pi, \pi)$$

$$= (\sqrt{2} - \pi, 2\sqrt{2} + \pi, 3\sqrt{2} - \pi)$$

added sheet

Example: Do two steps of Gauss-Jordan reduction on the matrix below but record the steps as linear combinations of rows.

$$\begin{bmatrix} 1 & 2 & 1 \\ -1 & 2 & 0 \\ 3 & 0 & 8 \end{bmatrix} \xrightarrow[\text{①}]{r_1 + r_2 \mapsto r_2} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 4 & 1 \\ 3 & 0 & 8 \end{bmatrix} \xrightarrow[\text{②}]{r_3 - 3r_1 \mapsto r_3} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 4 & 1 \\ 0 & -6 & 5 \end{bmatrix}$$

$$\text{①} \;\to \begin{bmatrix} r_1 \\ r_1 + r_2 \\ r_3 \end{bmatrix} \xrightarrow[\text{②}]{} \begin{bmatrix} r_1 \\ r_1 + r_2 \\ r_3 - 3r_1 \end{bmatrix}$$

Example: Write three distinct linear combination of the vectors $\overline{u}_1 = (1, 2, 3)$ and $\overline{u}_2 = (1, -1, 1)$.

Example: Is $\overline{v} = (2, -1, 2)$ a linear combination of $\overline{u}_1 = (1, 2, 3)$ and $\overline{u}_2 = (1, -1, 1)$?

Find c_1 and c_2 so that

$$c_1 \vec{u}_1 + c_2 \vec{u}_2 = (2,-1,2)$$

$$c_1(1,2,3) + c_2(1,-1,1) = (2,-1,2)$$

So

$$(c_1+c_2, 2c_1-c_2, 3c_1+c_2) = (2,-1,2)$$

So

$$\left.\begin{aligned} c_1 + c_2 &= 2 \\ 2c_1 - c_2 &= -1 \\ 3c_1 + c_2 &= 2 \end{aligned}\right]$$

SoLE to solve.

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & -1 & -1 \\ 3 & 1 & 2 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

As a SoLE, this has no solution. So...

Answer: $\vec{v}$ is NOT a linear combination of of $\vec{u}_1$ and $\vec{u}_2$.

True or False

If the matrix B is the reduced row echelon form of matrix A, then the rows of B are linear combinations of the rows of A.

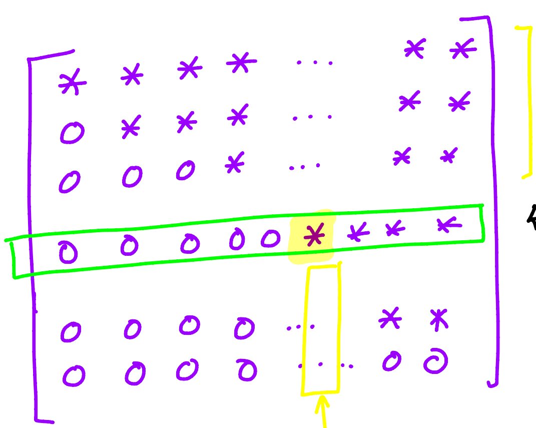
True!

Elementary row operations are linear combinations of the rows of the matrix.

In echelon form, no nonzero row can be a linear combination of any of the other nonzero rows.

True!

All of these would have to be multiplied by zero.



Can this be a linear combination of the other rows?

These are all zero.

There are no ways to get this entry.