We (or the book) proved that the following are vector spaces:

- $V_1 = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \ : \ y = 2x \right\}$ under regular vector addition and scalar multiplication.
- $V_2 = \{ f : \mathbb{R} \to \mathbb{R} \ : \ f(x) + 3f'(x) = 0 \}$ under regular function addition and scalar multiplication.
- $V_3 = \{ a_2x^2 + a_1x + a_0 \ : \ a_2, a_1, a_0 \in \mathbb{R} \}$ under regular polynomial addition and scalar multiplication.

Explain why the following examples are *not* vector spaces. Try to find as many reasons as you can.

1. $V = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \ : \ y = x + 2 \right\}$ under regular vector addition and scalar multiplication.

2. $V = \{ f : \mathbb{R} \to \mathbb{R} \ : \ f(x) + 3f'(x) = 1 \}$ under regular function addition and scalar multiplication.

3. $V = \{ a_2x^2 + a_1x + a_0 \ : \ a_2, a_1, a_0 \in \mathbb{Z} \}$ under regular polynomial addition and scalar multiplication. The symbol $\mathbb{Z}$ denotes all integers: $... - 2, -1, 0, 1, 2, 3, 4, ...$

Lemma 1.16 In any vector space V and for any $\vec{v} \in V$ and $r \in \mathbb{R}$,

Section Two.I.2: Subspaces and Spanning Sets

Definition:

Example: Let $V = \mathbb{R}^3$, the vector space of 3-dimensional real-valued vectors under the usual vector and scalar operations. Let $W = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \ : \ x + y - z = 0 \right\}$. Show W is a subspace of V.