SECTION TWO.I.2: LINEAR INDEPENDENCE

1. Below are several *subsets* of $V = \mathbb{R}^3$. Which ones span $\mathbb{R}^3$? Are some more efficient than others?

   (a) $A = \{(1, 1, 0), (0, 1, 0)\}$

   (b) $B = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$

   (c) $C = \{(1, 0, 0), (0, 1, 0), (0, 0, 1), (1, 1, 0)\}$

   (d) $D = \{(0, 1, 0), (0, 0, 1), (1, 1, 0)\}$

2. **Definition:** Let S be a subset of the vectors in the vector space V. We say S is **linearly independent** if

3. Determine if the set $T = \{\vec{u} = (1, 2, 0), \vec{v} = (1, 1, 1), \vec{w} = (1, 3, -1)\}$ of vectors in $\mathbb{R}^3$ are linearly independent.

4. Determine if the set $S = \{\vec{s_1} = (1,2,1,1), \vec{s_2} = (1,1,1,1), \vec{s_3} = (3,4,0,-1), \vec{s_4} = (0,8,-1,4)\}$ of vectors in $\mathbb{R}^4$ are linearly independent.

Lemma 1.5 $S = \{\vec{s_1}, \vec{s_2}, \vec{s_3}, \cdots, \vec{s_n}\}$ is a subset of the vector space V.

S is linearly independent

5. V is a vector space and $S \subseteq V$, $\vec{v} \in V$. What can you conclude if $[S \cup \{\vec{v}\}] = [S]$? Can your reverse this implication?

6. V is a vector space and $S \subseteq V$, $\vec{s} \in S$. What can you conclude if $[S - \{\vec{s}\}] = [S]$? Can you reverse this implication?

7. Let S be a subset of the vector space V. If, for every $\vec{v} \in S$, $[S - \vec{v}] \neq [S]$ (that is, the subspace $[S - \vec{v}]$ is smaller than the space $[S]$), what can you conclude about S? Does the reverse implication still hold?