1. (Warm-up) Let $S \subset V$ where V is a vector space and $S = \{\vec{s_1}, \vec{s_2}, \cdots \vec{s_n}\}$.

   (a) What can you say about the relationship between the objects $[S - \{\vec{s_i}\}]$, $[S]$, and V?

   $[S - \{\vec{s_i}\}] \subseteq [S] \subseteq V$

   (They are nested subspaces. They may be equal.)

   (b) What can you conclude if $[S - \{\vec{s_i}\}] \neq [S]$ for every $\vec{s_i} \in S$?

   So $[S - \{\vec{s_i}\}] \subsetneq [S]$. That is $[S - \{\vec{s_i}\}]$ is a strictly smaller subspace. So $\vec{s_i} \notin [S - \{\vec{s_i}\}]$. So S is linearly independent

* 2. **Definition:**

A basis of a vector space V is a $\boxed{\text{sequence}}$ of vector that are linearly independent and span V.

3. Which of the following sets form a *basis* for $\mathbb{R}^3$? (Note: These are the same sets of vectors from Monday's sheet.)

   (a) $A = \langle\, (1,1,0), (0,1,0) \,\rangle$

   not a basis. doesn't span

   (b) $B = \langle\, (1,0,0), (0,1,0), (0,0,1) \,\rangle$

   basis

   (c) $C = \langle\, (1,0,0), (0,1,0), (0,0,1), (1,1,0) \,\rangle$

   not a basis.
   not linearly independent.

   (d) $D = \langle\, (0,1,0), (0,0,1), (1,1,0) \,\rangle$

   basis

4. Write the vector with coordinates $(1, -2, 3)$ using each basis below:

(a) $B_1 = \langle\, (1,0,0), (0,1,0), (0,0,1) \,\rangle$

$\vec{s}_1 \quad \vec{s}_2 \quad \vec{s}_3$

$$(1,-2,3) = 1\vec{s}_1 + (-2)\vec{s}_2 + 3\vec{s}_3$$

$$\mathrm{rep}_{B_1}\big((1,-2,3)\big) = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}_{B_1}$$

$\vec{s}_1 \quad \vec{s}_2 \quad \vec{s}_3$

(b) $B_2 = \langle\, (0,1,0), (0,0,1), (1,1,0) \,\rangle$

$$(1,-2,3) = -3\cdot\vec{s}_1 + 3\vec{s}_2 + 1\cdot\vec{s}_3$$

$$\mathrm{rep}_{B_2}\big((1,-2,3)\big) = \begin{bmatrix} -3 \\ 3 \\ 1 \end{bmatrix}_{B_2}$$

(c) $B_3 = \langle\, (1,1,0), (0,1,0), (0,0,1) \,\rangle$

$$\mathrm{rep}_{B_3}\big((1,-2,3)\big) = \begin{bmatrix} 1 \\ -3 \\ 3 \end{bmatrix}_{B_3}$$

5. Write the vector $1 - 2x + 3x^2$ with respect to the basis $B = \langle\, 1, x, x - x^2 \,\rangle$.

$$\mathrm{rep}_B\big(1-2x+3x^2\big) = \begin{bmatrix} 1 \\ 1 \\ -3 \end{bmatrix}_B$$

Check: $1(1) + 1\cdot(x) - 3(x - x^2) = 1 + x - 3x + 3x^2$
$= 1 - 2x + 3x^2$