SECTION TWO.III.1: BASIS

1. (Warm-up) Let $S \subset V$ where V is a vector space and $S = \{\vec{s_1}, \vec{s_2}, \cdots \vec{s_n}\}$.

   (a) What can you say about the relationship between the objects $[S - \{\vec{s_i}\}]$, $[S]$, and V?

   (b) What can you conclude if $[S - \{\vec{s_i}\}] \neq [S]$ for every $\vec{s_i} \in S$?

2. **Definition:**

3. Which of the following sets form a *basis* for $\mathbb{R}^3$? (Note: These are the same sets of vectors from Monday's sheet.)

   (a) $A = \langle\, (1,1,0), (0,1,0) \,\rangle$

   (b) $B = \langle\, (1,0,0), (0,1,0), (0,0,1) \,\rangle$

   (c) $C = \langle\, (1,0,0), (0,1,0), (0,0,1), (1,1,0) \,\rangle$

   (d) $D = \langle\, (0,1,0), (0,0,1), (1,1,0) \,\rangle$

4. Write the vector with coordinates $(1, -2, 3)$ using each basis below:

   (a) $B_1 = \langle\, (1,0,0), (0,1,0), (0,0,1) \,\rangle$

   (b) $B_2 = \langle\, (0,1,0), (0,0,1), (1,1,0) \,\rangle$

   (c) $B_3 = \langle\, (1,1,0), (0,1,0), (0,0,1) \,\rangle$

5. Write the vector $1 - 2x + 3x^2$ with respect to the basis $B = \langle\, 1, x, x - x^2 \,\rangle$.