Characteristic Equation

1. **Example:** Find the eigenvalues associated with the matrix $A = \begin{pmatrix} 4 & 1 \\ -8 & -5 \end{pmatrix}$

2. **Definition/Lemma 3.9 §5.2.3:** A scalar λ is an eigenvalue of an $n \times n$ matrix A if and only if λ satisfies the *characteristic equation* $det(A - \lambda I_n) = 0$. The expression $det(A - \lambda I_n)$ is called the *characteristic polynomial* of A.

3. **Example:** Find the characteristic polynomial and all eigenvalues associated with the matrix $B = \begin{pmatrix} -2 & 3 & 4 \\ 0 & 5 & 1 \\ 0 & 0 & 1 \end{pmatrix}$.

4. **Example:** Construct a 4×4 matrix with eigenvalues $0, -1, 7, 7$.

5. **Example:** Are the previous examples singular or nonsingular matrices?

6. **Recall:** Interchanging rows of a matrix changes its determinant by

   Replacing a row with itself plus a multiple of an another row changes the determinant

7. **Observation:** It is possible to take any square matrix A, use *only* the two row operations from item #6, and find an upper-triangular row equivalent matrix B such that

$$det(A) = (-1)^r \det(B) = (-1)^r \cdot (\text{product of the main diagonal of } B),$$

where r is the number of row exchanges.

8. **Example:** Assume $B = \begin{pmatrix} -2 & 3 & 4 \\ 0 & 5 & 1 \\ 0 & 0 & 1 \end{pmatrix}$ is obtained form A by the following row operations: (i) exchange rows 1 and 3; (ii) add $2\cdot$(row 1) to row 3; (ii) add $-2\cdot$ row 1 to row 2. Find $det(A)$.

9. **Theorem:** The square matrix A is nonsingular if and only if

10. **Definition 1.2 §5.2.1:** Let A and B be $n \times n$ matrices. We say A **is similar to** B if there exists an invertible matrix P such that $P^{-1}AP = B$.

11. **Example:** Show that $A = \begin{pmatrix} -13 & 21 \\ 7 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 8 & 0 \\ 0 & -20 \end{pmatrix}$ are similar. (Hint: Use $P = \begin{pmatrix} 1 & 3 \\ 1 & -1 \end{pmatrix}$.