EIGENVALUES AND EIGENVECTORS

1. **Definition 3.5 §5.2.3:** An **eigenvector** of an $n \times n$ matrix A is a nonzero vector $\vec{v}$ such that $A\vec{v} = \lambda\vec{v}$. A scalar λ is called an **eigenvalue** of A if there exists a nontrivial solution to the equation $A\vec{x} = \lambda\vec{x}$. In this case, we say $\vec{x}$ is an eigenvector associated with eigenvalue λ.

2. Example 1: Let $A = \begin{pmatrix} 1 & 6 \\ 5 & 2 \end{pmatrix}$.

   (a) Show that $\vec{v} = \begin{pmatrix} 3 \\ -5/2 \end{pmatrix}$ is an eigenvector but $\vec{w} = \begin{pmatrix} 3 \\ -5 \end{pmatrix}$ is not.

   (b) Show that 7 is an eigenvalue of A.

3. **Definition 3.1.2 §5.2.3:** For $n \times n$ matrix A. The set of all eigenvectors associated with eigenvalue λ forms a subspace of $\mathbb{R}^n$ and is called the **eigenspace** associated with eigenvalue λ.

4. Example 2: Let $A = \begin{pmatrix} -1 & -2 & 5 \\ 2 & -5 & 5 \\ 2 & -2 & 2 \end{pmatrix}$, a matrix with eigenvalue -3. Find a basis for the corresponding eigenspace.

5. **Theorem:** If A is triangular, then its eigenvalues are the entries on its main diagonal.

6. Example: Construct a 3×3 matrix A with eigenvalues $0, -1,$ and 5 and find an eigenvector associated with each eigenvalue.

7. **Theorem 3.18** §**5.2.3:** Let A be an $n \times n$ matrix with eigenvectors $\vec{v_1}, \vec{v_2}, \cdots, \vec{v_k}$ associated with distinct eigenvalues $\lambda_1, \lambda_2 \cdots, \lambda_k$. Then the set of eigenvectors $\{\vec{v_1}, \vec{v_2}, \cdots, \vec{v_k}\}$ is linearly independent.

8. Question: Does the previous theorem really need the eigenvalues to be distinct?

9. Question: If $\vec{v}$ is an eigenvector of matrix A associated with eigenvalue λ, can you draw any conclusions about eigenvectors and/or eigenvalues for matrix A^2?

10. Question: How would you go about finding *all* eigenvalues associates with matrix A.?