1. List all distinct polynomials of degree 3 or less in $\mathbb{Z}_2$.

2. Compute each of the following if $p(x) = 5x^2 + 3x - 4$ and $q(x) = 4x^2 - x + 9$ are polynomials in $\mathbb{Z}_{12}$.

   (a) $p(x) + q(x)$

   (b) $p(x) \cdot q(x)$

   (c) $(p(x))^2$

3. Use the Division Algorithm to find $q(x)$ and $r(x)$ such that $a(x) = q(x)b(x) + r(x)$ where $\deg(r(x)) < \deg(b(x))$ or $r(x)$ is the zero polynomial.
   For this problem, it is OK to just state your $q(x)$ and $r(x)$, but you want to make sure you know how to find them. To format the division algorithm in LaTeX is unnecessarily tedious.

   (a) $a(x) = 5x^3 + 6x^2 - 3x + 4$, $b(x) = x - 2$, in the polynomial ring $\mathbb{Z}_7[x]$

   (b) $a(x) = x^5 + x^3 - x^2 - x$, $b(x) = x^3 + x$, in the polynomial ring $\mathbb{Z}_2[x]$

4. Final all zeros for each of the following polynomials or demonstrate that none exist.

   (a) $5x^3 + 4x^2 - x + 9$ in $\mathbb{Z}_{12}[x]$ (Hint: There is an easier way than just testing all 12 possibilities!

   (b) $5x^4 + 2x^2 - 3$ in $\mathbb{Z}_7[x]$

   (c) $x^5 + x^3 + 1$ in $\mathbb{Z}_2[x]$

5. Find a polynomial $p(x)$ in $\mathbb{Z}_4$ of degree at least 2 such that $p(x)$ is a unit.

6. Give two different factorizations of $x^2 + x + 8$ in $\mathbb{Z}_{10}[x]$.

7. (a) Give an example of polynomials $a(x)$ and $b(x)$ in $\mathbb{Z}[x]$ such that the Division Algorithm fails.

   (b) Explain what hypothesis in Theorem 17.6 fails to hold for the polynomials and/or the ring from part (a).