1. For every polynomial of degree 2or 3 in $\mathbb{Z}_2$ (see the table below), either factor it completely or state that it is irreducible.

polynomial	factored form	zeros
x^2	$x \cdot x$	$x = 0$
$x^2 + x$		
$x^2 + 1$		
$x^2 + x + 1$		
x^3		
$x^3 + x^2$		
$x^3 + x^2 + x$		
$x^3 + x^2 + 1$		
$x^3 + x^2 + x + 1$		
$x^3 + x$		
$x^3 + x + 1$		
$x^3 + 1$		

2. Prove or Disprove: For every $n \in \{1,2,3,4,5\}$ there exists a polynomial $p(x)$ of degree n with more than n **distinct** zeros in $\mathbb{Z}_6$.

degree	polynomial	zeros
1		
2		
3		
4		
5		

3. Let $I = \langle x^3 + 2x^2 \rangle$ be an ideal in $\mathbb{Q}[x]$.

   (a) List 5 distinct elements of $\mathbb{Q}[x]$ that are in I, list 5 distinct elements of $\mathbb{Q}[x]$ that are **not** in I, and then describe in words or symbols what the ideal I looks like.

   (b) Is I a maximal ideal? Justify your answer.

4. Suppose $f(x)$ is irreducible in $F[x]$, where F is a field. Prove that for every nonzero polynomial $g(x) \in F[x]$, either $\gcd(f(x), g(x)) = 1$ or $f(x) \,\big|\, g(x)$.

5. (a) Rewrite Fermat's Little Theorem as stated in your text book.

   (b) Show that Fermat's Little Theorem applies for $n = 5$, by explicitly calculating a^4 for every $a \in \{1,2,3,4\}$ and checking that each is congruent to 1.

(c) Explain what Fermat's Little Theorem implies about factorizations of $x^p - x$ in $\mathbb{Z}_p[x]$ where p is a prime. Prove that your assertion is correct.

6. Show that $\alpha = \sqrt{5} + \sqrt{3}i$ is algebraic over $\mathbb{Q}[x]$ by explicitly finding $p(x) \in \mathbb{Q}[x]$ such that $p(\alpha) = 0$.