

For each problem below,  $G$  is a group with subgroups  $H$  and  $K$ .

1. Prove that  $H \cup K$  is not necessarily a group of  $G$ .

**Answer:**

2. Prove that  $H \cap K$  is a subgroup of  $G$ .

**Proof:**

3. Let  $HK = \{hk : h \in H \text{ and } k \in K\}$ . Prove that  $HK$  is not necessarily a subgroup of  $G$ .

**Proof:**

4. Let  $HK = \{hk : h \in H \text{ and } k \in K\}$ . Prove that if  $G$  is **abelian**, then  $HK$  is a subgroup of  $G$ .

**Proof:**