1. Let $\psi : G \to H$ be a group homomorphism. Prove that $\psi$ is one-to-one if an only if $\psi^{-1}(e_H) = e_G$.
   **Hint:** You should use the First Isomorphism Theorem to prove one direction of the if and only if statement.

   **Proof:**

2. Prove that if $G$ is a finite group and $\phi : G \to H$ is a group homomorphism, then $|\phi(G)|$ divides both $|G|$ and $|H|$.

   **Proof:**

3. Find all of the homomorphisms from $\phi : \mathbb{Z} \to \mathbb{Z}$. Justify your answer.
   **Hint:** For this problem and the next, you may want to use some facts you proved in HW 8.

   **Answer:**
   **Justification:**

4. (a) Find all of the automorphisms of $\mathbb{Z}_8$ and explain your reasoning.

   **Answer with explanation:**

   (b) Prove that $\text{Aut}(\mathbb{Z}_8) \cong U(8)$. Recall that $\text{Aut}(G)$ is the group of automorphisms of $G$ under the operation of function composition.

   **Proof:**

5. (a) Find all abelian groups of order $n$ for $n \in \{15, 16, 17, 18, 19, 20\}$.

   **Answer:**

   (b) For each nonisomorphic group of order 18, find an element of order 3.

   **Answer:**

6. Which of the following sets are rings? If it is a ring, does it have a multiplicative identity? Is it commutative? An integral domain? A division ring? What are its units, if any? Is it a field? (You decide your explanations. Speak to your future self!)

   (a) $7\mathbb{Z}$ with usual addition and multiplication

   | question | answer |
   |---|---|
   | **a ring?** | Answer here! |
   | **with unity?** | Answer here! |
   | **commutative?** | Answer here! |
   | **an integral domain?** | Answer here! |
   | **its units?** | Answer here! |
   | **a division ring?** | Answer here! |
   | **a field?** | Answer here! |

   (b) $\mathbb{Z}_7$ with usual addition and multiplication

   | question | answer |
   |---|---|
   | **a ring?** | |
   | **with unity?** | |
   | **commutative?** | |
   | **an integral domain?** | |
   | **its units?** | |
   | **a division ring?** | |
   | **a field?** | |

   (c) $\mathbb{Z}_{18}$ with usual addition and multiplication

   | question | answer |
   |---|---|
   | **a ring?** | |
   | **with unity?** | |
   | **commutative?** | |
   | **an integral domain?** | |
   | **its units?** | |
   | **a division ring?** | |
   | **a field?** | |

   (d) $\mathbb{Q}(\sqrt{2}) = \{a + b\sqrt{2} : a, b \in \mathbb{Q}\}$ with usual addition and multiplication

   | question | answer |
   |---|---|
   | **a ring?** | |
   | **with unity?** | |
   | **commutative?** | |
   | **an integral domain?** | |
   | **its units?** | |
   | **a division ring?** | |
   | **a field?** | |

(e) $R = \{a + b\sqrt[3]{2} : a, b \in \mathbb{Q}\}$ with usual addition and multiplication

| question | answer |
|---|---|
| **a ring?** | |
| **with unity?** | |
| **commutative?** | |
| **an integral domain?** | |
| **its units?** | |
| **a division ring?** | |
| **a field?** | |

(f) $\mathbb{M}_2(\mathbb{Z}_2)$, the set of $2 \times 2$ matrices with entries from $\mathbb{Z}_2$ with usual matrix addition and multiplication

| question | answer |
|---|---|
| **a ring?** | |
| **with unity?** | |
| **commutative?** | |
| **an integral domain?** | |
| **its units?** | |
| **a division ring?** | |
| **a field?** | |

(g) $\mathbb{R}_1[x] = \{ax + b : a, b \in \mathbb{R}\}$, the set of linear polynomials with real coefficients with the usual addition and multiplication

| question | answer |
|---|---|
| **a ring?** | |
| **with unity?** | |
| **commutative?** | |
| **an integral domain?** | |
| **its units?** | |
| **a division ring?** | |
| **a field?** | |

(h) $\mathbb{R}_\infty[x]$ the set of all polynomials with real coefficients with the usual addition and multiplication

| question | answer |
|---|---|
| **a ring?** | |
| **with unity?** | |
| **commutative?** | |
| **an integral domain?** | |
| **its units?** | |
| **a division ring?** | |
| **a field?** | |