NAME:

**Instructions:** Answer questions in the space provided. You will be graded both on correctness and presentation. This test has 7 questions worth a total of 100 points.

1. (3 points each) Give examples of the following, if they exist. Otherwise briefly explain why such examples do not exist.

   (a) Two nonisomorphic groups of order 20.

   (b) An infinite nonabelian group.

   (c) A group with two distinct subgroups of order 5.

   (d) A nonabelian group of order 11.

   (e) An element of order 10 in $S_7$.

   (f) An element of order 10 in $A_7$.

2. (12 points) Consider the permutation group $S_8$, and let $\alpha = (1235)(24567)(1572)$.

(a) Express $\alpha$ as a product of disjoint cycles.

(b) What is the inverse of $\alpha$?

(c) What is the order of $\alpha$?

(d) Is $\alpha$ an even or odd permutation?

3. (10 points)

(a) State the definition of an automorphism of a group $G$.

(b) Find the group of automorphisms of the cyclic group $Z_{18} = \langle a \rangle$.

(c) List all subgroups of $Z_{18}$.

4. (10 points)

   (a) Show that $U(21)$ is not isomorphic to $\mathbb{Z}_{12}$.

   (b) Show that $D_6$ is not isomorphic to $A_4$.

5. (20 points)

(a) State the definition of a *group isomorphism*.

(b) Define the following set of matrices:

$$G = \left\{ \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \;\middle|\; a \in \mathbb{Z} \right\}.$$

The set $G$ under matrix multiplication is a group. (You don't need to prove that $G$ is a group.) Prove that $G$ is isomorphic to $\mathbb{Z}$.

6. (20 points)

(a) State the definition of a *group*.

(b) Let $S = \{x \in \mathbb{R} \mid x \neq 0\}$ be the set of nonzero real numbers, and define a binary operation on $S$ by the formula $a \star b = 2ab$. Is $S$ with this binary operation a group? Prove or disprove.

7. (10 points) Prove that for any group $G$ and any $a, b \in G$, $|ab| = |ba|$.