Name: ____________________

Rules:

You have 1.5 hours to complete this midterm.

Partial credit will be awarded, but you must show your work.

No notes, books, or other aids are allowed.

Turn off anything that might go beep during the exam.

Good luck!

Problem	Possible	Score
1	16	
2	20	
3	16	
4	12	
5	12	
6	24	
Extra Credit	5	
Total	100	

1. Let G and H be groups and let $\phi : G \to H$ be a group homomorphism.

   (a) (2 pts) State the definition of a **group homomorphism**.

   (b) (2 pts) State the definition of the **kernel of** ϕ, ker ϕ.

   (c) (12 pts) Prove ker ϕ is a normal subgroup of G. (Note that you must show ker ϕ is a subgroup of G **and** that it is normal.)

2. (20 points) Give an examples of the following, if they exist. Otherwise briefly explain why such examples do not exist.

   (a) A commutative ring with unity that is not an integral domain.

   (b) A ring that is an integral domain but is not a field.

   (c) A ring R and a nontrivial subring I such that I is an ideal of R

   (d) A ring R and a nontrivial subring S such that S is **not** an ideal of R

   (e) A ring R and an ideal I that is prime.

3. (16 points)

   (a) (4 pts) State the First Isomorphism Theorem (for groups)

   (b) (12 pts) Let $\psi : G \to H$ be a group homomorphism. Prove that ψ is one-to-one if and only if $\psi^{-1}(e_H) = \{e_G\}$.

4. (12 points) Prove that if R is a field, the only ideals of R are $\{0\}$ and R itself.

5. (12 points) Let R be a ring and let $a \in R$. Prove that the set $S = \{r \in R : ra = 0\}$ is a subring of R. Note that you should not assume R is commutative.

6. (24 points)

   (a) List all nonisomorphic abelian groups of order 24.

   (b) Let $\mathbb{R}$ be the ring of real numbers under the usual operations of addition and multiplication. Explain why the function $f : \mathbb{R} \to \mathbb{R}$ defined as $f(x) = 2x + 1$ is not ring homomorphism.

   (c) Find all group homomorphisms from $\mathbb{Z}_{16}$ to $\mathbb{Z}_{18}$. Your answer(s) must be stated as functions.

   (d) Give a maximal ideal in the ring $\mathbb{Z}_{20}$

Extra Credit: (5 points) Prove that every finite integral domain is a field.